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HOW ONE CAN INTRODUCE COMPLIANCE INTO COMPUTER MODELS OF THE MULTIBODY DYNAMICS USING FEATURES OF OBJECT-ORIENTED MODELING

IVAN I. KOSENKO[*] AND MIKHAIL E. STAVROVSKIY[†]

[*] Moscow Aviation Institute (National Research University)
Volokolamskoe shosse 4, 125993, Moscow, Russia
e-mail: kosenko@ccas.ru
http://www.ccas.ru/depart/mechanics/TUMUS/Kosenko/Kosenko.html

[†] Research Center on Problems for Management of Resource Saving and Waste
Olympic avenue 42, 141006, Mytishi, Moscow region, Russia
e-mail: stavrov@mail.ru

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Summary. *Dynamics of a multibody system is simulated in a most universal way in case of contacts having a compliance property. This latter case is implemented usually by elasticity / viscosity along the direction normal to the rigid body outer / inner surface and by friction along its tangent direction. The Hertz model is one of the most popular elastic contact models for engineering applications. Object-oriented approach for building up the multibody dynamics model simulating compliant contacts is under development in this paper. A technology for constructing classes-templates is applied to build up contact objects in the dynamical model. The Hertz contact model is under consideration as a simplest example.*

1 INTRODUCTION

A lot of methods for describing a structure of the multibody system using different graph approaches are known. See for instance [1, 2, 3], and further references one can find there. Usually multibody system is assumed to consist of rigid bodies. Note that in frame of the bond graph approach a background of energy exchanges is used [4].

Either directed or undirected graphs are used depending on the problems to be resolved for implementing the multibody structural analysis based on the force interactions. Using known Newton's laws [5] one can approach the dynamics from so called Newton's viewpoint. In such a way the translational-rotational motion of each body is described by the system of Newton – Euler's ODEs. The multibody system graph structure is constructed using an analysis for mutual interactions of bodies the system composed from. Such interactions are caused mainly by mechanical constraints. In general, Newton's third law of dynamics implies a mutual nature of interactions between the bodies thus causing the system graph to be undirected.

In some particular cases the graph can have a special structure, like it takes place for holonomic constraints composing the system of tree structure. Such a situation occurs for instance in robotics where the tree structure is used to reduce the source Newton – Euler system of ODEs with an attached subsystem of algebraic equations to some special kind of dynamical equations, like ones of Lagrange of the second kind.

In general case the situation is more complicated, especially if non-holonomic constraints are used. In any way one has to take into account equations of constraints being attached to dynamical ODEs. One can mention that there exists a background for building up models of a type mentioned above using: algorithms [6], modeling languages [7], and compilers [8]. To describe the model of the multibody system under consideration one starts from: (a) an object-oriented paradigm [9] on one hand, and (b) so called physical principles of modeling [10] on the other one.

2 DESCRIPTION USING UNDIRECTED GRAPH

Thus assume the multibody system consists of $m + 1$ bodies $B_0, \ldots, B_m$, see Figure 1. Represent them as composing a finite set $\mathcal{B} = \{B_0,\ldots, B_m\}$. Here B_0 is assumed to be a base body. The body B_0 is assumed to be connected with an inertial frame of reference, or to have a known motion with respect to the inertial frame of reference. One can represent the base body, for example, as a rotating platform, or as a vehicle performing known predefined motion.

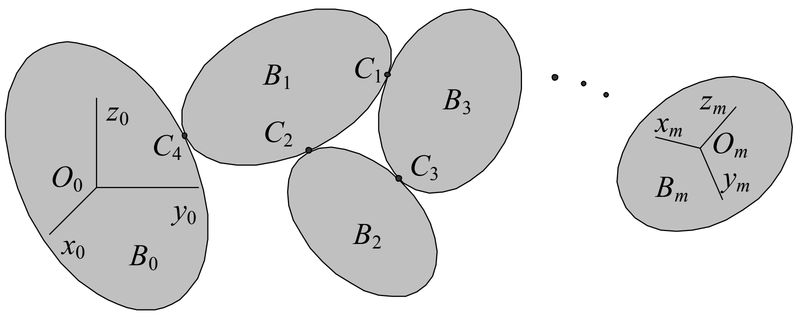

Figure 1: Multibody system

Some bodies are possible to be interconnected mechanically by constraints. Assume that all constraints compose the set $\mathcal{C} = \{C_1,\ldots, C_n\}$. We include in our considerations constraints of the following types: holonomic / nonholonomic, scleronomic / rheonomic, and, what essential, mechanical contacts with compliances.

As a result one can uniquely represent a structure of the multibody system by an undirected graph $G = (\mathcal{B}, \mathcal{C}, \mathcal{I})$. Here $\mathcal{I} \subset \mathcal{C} \times \mathcal{B}$ is an incidence relation setting in a correspondence for every edge $C_i \in \mathcal{C}$ of the graph the vertex $B_k \in \mathcal{B}$ incident to it. According to physical reasons it is easy to see that for any mechanical constraint C_i there exist exactly two bodies / vertices $B_k, B_l \in \mathcal{B}$ being connected by this constraint.

The incidence relation generates an adjacency relation $\mathcal{S} \subset \mathcal{B} \times \mathcal{B}$ on the set of vertices. In our case this relation has the properties: (a) antireflexiveness, a body cannot be connected with itself; (b) symmetry, because of the graph is undirected: if $(B_k, B_l) \in \mathcal{S}$, then $(B_l, B_k) \in \mathcal{S}$.

3 DESCRIPTION USING COMMUNICATION NETWORK

Evidently, one has insufficient structural information derived from graph G for describing the multibody system dynamics. Indeed, in addition to the force interaction which is represented usually by wrenches between bodies B_k, B_l via the constraint C_i one has also a kinematical conditions specific for constraints of different kind. Wrenches acting between interconnected bodies themselves are mutually interconnected by virtue of Newton's third law of dynamics because one can represent these wrenches in turn by constraint forces, and constraint couples.

Thus if one can associate the system of ODEs for translational-rotational motion with the object of a model corresponding to the rigid body, then one can in a natural way associate the system of algebraic equations with the object of a model corresponding to constraint. Note that according to consideration has been done above the set of an algebraic equations is composed of equations for constraint forces (reactions), torques of couples (also reactions), and kinematical equations attached depending on a type of the constraint. So in a way outlined above the differential and algebraic equations are said encapsulated in behavioral sections of objects representing rigid bodies and constraints respectively.

Furthermore, one can reduce any multibody system dynamical model to two subsets of objects: subset of rigid body models (objects $B_0,\ldots, B_m$), and subset of models for constraints (objects $C_1,\ldots, C_n$). According to such an approach simulation of the whole system behavior is reduced to the permanent information interchange between objects of these two types have been enumerated above. Based on Newton's laws of dynamics one can build up the communication network implementing such an interchange.

Information channels connecting objects of two classes from above can also be classified into two types of ports: (a) class for the wrench transportation consisting of the force, torque, and point of the force application; (b) class for the twist transportation consisting of the mass center velocity, rigid body angular velocity plus additional auxiliary kinematical data. In our idealized model force interaction between bodies is performed at a geometric point. Its coordinates are exported to communicative network through the wrench port mentioned permanently in time.

Metaphorically an object of the rigid body class works as follow: it accepts data of wrenches acting to the rigid body and exports kinematical data of this body. At the same time and permanently the constraint object imports kinematical data of two interconnected rigid bodies and exports data of wrenches acting in directions of these two rigid bodies under constraint mentioned and being assumed to be generated by the constraint of specific type. As a result all these objects interact like total communication network permanently in time. The library of classes performing functionality under description here is implemented in frame of Modelica language [7].

One must consider all connections used as bidirected ones. This property corresponds to the undirectional case for the multibody graph edge. Real direction of the causality „flow“ is usually defined by the compiler. Such style of the model development is called an acausal modeling [8].

4 DESCRIPTION USING MULTIBOND GRAPH

Let us trace now the similarities between the bond graphs [4] and our models of the multibody systems. Power is a fundamental property of any (multi) bond graph. To be exact power is associated with or is conducted by the particular multibond at any current instant. Regarding our case consider the power of the forces acting upon the rigid body. Let the rigid body kinematics be defined by the twist (**v**, **ω**), where **v** is the mass center velocity, and **ω** is the rigid body angular velocity. Further let all the forces acting upon the body be reduced to the wrench (**F**, **M**) with the total force **F** and the total torque **M** being reduced to the mass center. Thus the total power of all the forces acting on the body is computed by the known formula: $W = (\mathbf{v}, \mathbf{F}) + (\boldsymbol{\omega}, \mathbf{M})$. This is exactly the same formula using for representing a multibond in the bond graph simulating the multibody system dynamics. We have thus an evident canonical duality between twists and wrenches.

Further, the pair of classes simulating twist and wrench plays a role of the multiport notion, and corresponding pairs of connections in visual model represent a notion of the multibond. One can associate in this way an object of the rigid body class with 1-junction, while 0-junction is associated with the object of a constraint class. The relevant general multibond graph representation of a constraint implementing compliant contact in any multibody system may be depicted as shown in Figure 2.

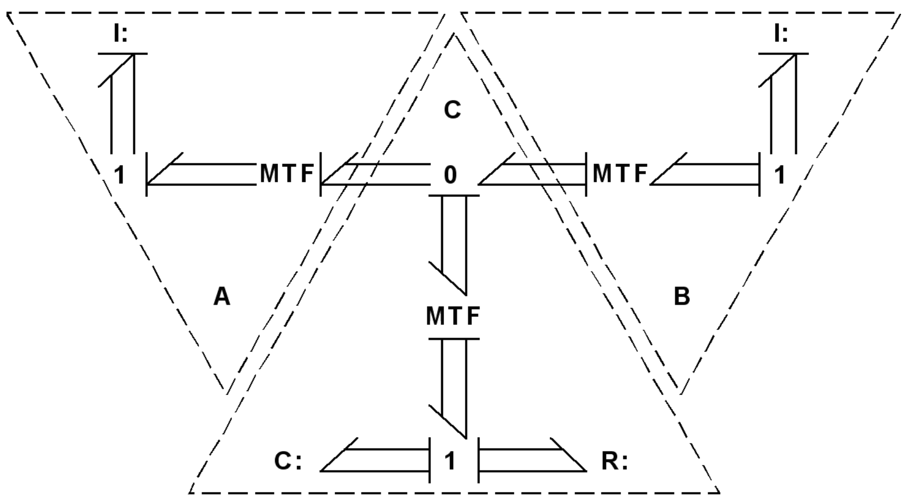

Figure 2: Multibond graph of constraint with compliance

Each multibond here consists of the twist (**v**, **ω**) signals representing the flow component and the wrench (**F**, **M**) signals as an effort. Causality of the inertance element arranges according to the Newton – Euler system of ODEs. Left and right transformers are to shift the twist from the mass center to the contact point according to the known Euler formula: $(\mathbf{v}, \boldsymbol{\omega}) \mapsto (\mathbf{v} + [\boldsymbol{\omega}, \mathbf{r}], \boldsymbol{\omega})$ where the vector **r** begins at the corresponding mass-center and ends at the point of contact. Reciprocally the wrenches shift to the body mass center from point of the contact in a following way: $(\mathbf{F}, \mathbf{M}) \mapsto (\mathbf{F}, \mathbf{M} + [\mathbf{r}, \mathbf{F}])$. As one can see easily transformers conserve power.

Central transformer in Figure 2 is responsible for the transfer to orthonormal base at the contact point with the common normal unit vector and two other base unit vectors being tangent ones to both contacting bodies' surfaces. These surfaces supposed regular enough. For certainty we interpret here the case of usual contact interconnection between the bodies by their outer / inner surfaces. If the inertial coordinates of these vectors compose columns of the orthogonal matrix of rotation Q then for shift from bottom to top across the transformer in Figure 2 one has the relation for the flow signals: $(\mathbf{v}, \boldsymbol{\omega}) \mapsto (Q\mathbf{v}, Q\boldsymbol{\omega})$. Likewise when shifting in a reverse direction one has a transformation of the efforts: $(\mathbf{F}, \mathbf{M}) \mapsto (Q^{-1}\mathbf{F}, Q^{-1}\mathbf{M})$ also conserving the power. Organization of the 0-junction depicted in Figure 2 provides a possibility for computing exactly the relative velocities at the constraint contact point.

Note that usually inertance elements are attached just to 1-junctions of bond graphs. This is because of causality issues in the models. Note also that Figure 2 can remind us in some degree an element of the lumped model for the flexible beam dynamics. On the other hand if we will act in a manner close to the real cases of constraints with the compliance then one has to use a compliant element with the explicit causality thus uniquely determined, see Figure 2, instead of the usual constraint elements. For details see the paper [11].

5 IMPLEMENTATION OF THE MECHANICAL CONTACT GENERAL MODEL

An experience of developing the models for elastic contacting of rigid bodies interactions in the multibody dynamics shows that an object-oriented facility, namely class templates, see Figure 3, provided by Modelica can be used to utilize a wide variety of different properties concerning a contact of solids. The properties are mainly from the following list: 1) geometric properties for surfaces in vicinity of the contact patch: (a) equations defining surfaces, (b) their gradients, (c) the Hesse matrices; 2) a model for computing the contact area dimensions and normal elastic force; 3) model for the normal viscous force of resistance; 4) model for the tangent forces along the tangent plane normal to elastic force.

A submodel of the geometry properties is to describe analytically algebraic surfaces of the structure complex enough [12]. For implementing the normal force computation one can choose now from at least two approaches: the Hertz model, or its volumetric modification [12]. Force of viscous resistance can also be modeled in several different ways: linear, non-linear, one proposed by Lankarani and Nikravesh [13], etc. In the models for tangent forces one can adopt either “simplest” approaches based on the Coulomb friction or more complex ones represented by the Contensou–Erismann [14, 15] model, or other models.

Class parameterization implemented in Modelica is the facility in line to apply to the problem under description. In our case we have four class parameters corresponding to the submodel categories enumerated above. An example to construct specific contact interaction model see in Figure 3. The example includes two stages of inheritance:

1) to derive a template with the force models, namely: the Hertz model for normal force, non-linear viscous force, the Contensou–Erismann model for the tangent dry friction forces;

2) to complete the whole construct one should define a specific geometry submodel for surfaces in contact.

6 CONCLUSIONS

For ellipsoid over the horizontal surface the Hertz model and the volumetric one were compared thoroughly in frame of wide range for different regimes of the ellipsoid motion. Ball bearing model has been verified for different models of elastic contact. The Contensou – Erismann model has been verified using known dynamics of the Tippe-Top.

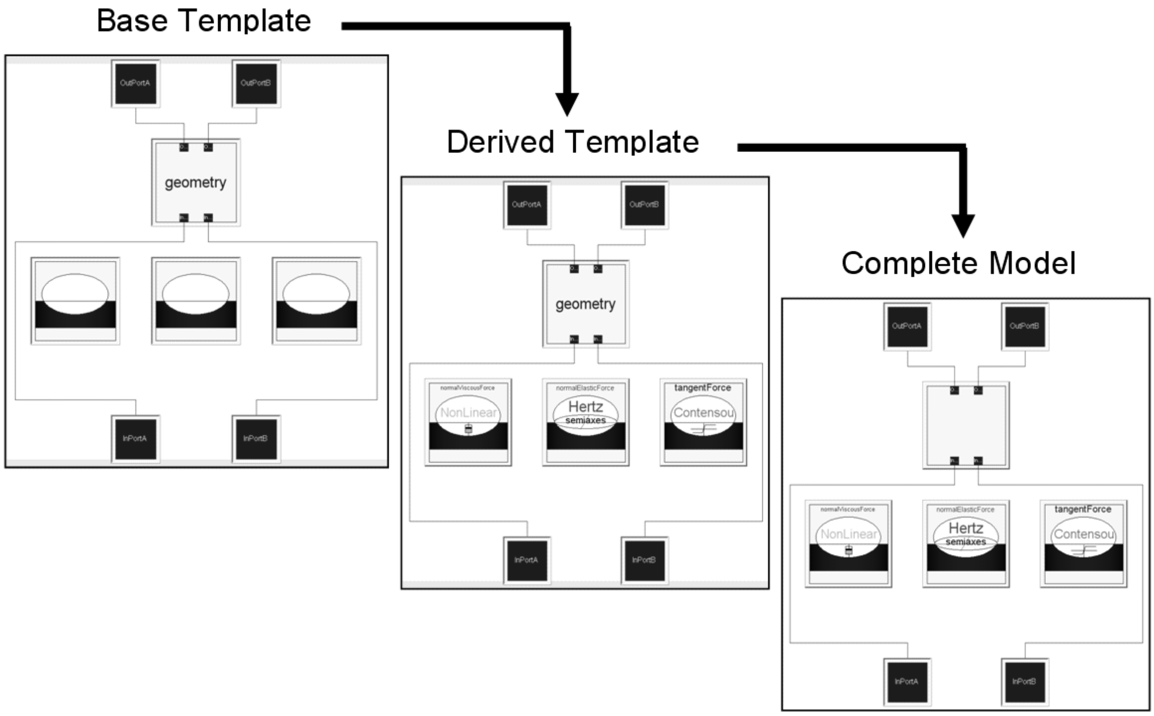

Figure 3: Stages for building up the contact model using templates

One can note the following main conclusions of the paper:

1. Physically-oriented approach for the multibody system dynamics simulation becomes quite easy and efficient, especially if we are going to build up the so-called virtual prototype.

2. An acausal modeling may be efficient and save a lot of time for the system development.

3. It turns out there exists a transparent correspondence between multiport, as a communication network, representation of the multibody dynamics model and its multibond graph representation.

4. Computational experience shows that the differential formulation, if applied properly, of contact tracking algorithm is more preferable than the traditional algebraic or even transcendental formulations. Its differential version becomes more reliable.

5. It turns out that evaluation of the complete elliptic integrals using subsystem of ODEs also has proved to be useful for the Hertz contact model: the computational models have become more reliable and fast.

6. The volumetric algorithm is more reliable and suitable for wide range of the contact area eccentricities, and on the other hand it provides the accuracy of 0.5% compared to the Hertz point algorithm.

7. The Tippe-Top “on head” revolution effect is caused by the dry friction force being distributed over the contact non-zero patch area.

8. For the isotropic case (circular contact area) the average values of the tangent forces for the Coulomb and Contensou–Erismann models are almost identical. But in the anisotropic case the first model becomes inadequate while the second one continues to be correct for the contacting process simulation. This property is important, for example, for a ball bearing simulation where the contact areas are essentially elliptic.

7 ACKNOWLEDGEMENT

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